Attenuation of shock wave from a point blast in a dusty air

Abstract. The behavior of shock wave, initiated by a point blast in a dusty air, is investigated. It is shown, that jump of parameters across shock front in dusty air follows another regularities, than in the case of ideal gas, beginning from the length of three dynamic radii, so, at ten dynamic radii difference can exceed 60%. When the heterogeneity of air is taken into account, substantial gradual changes of wave profile come over, and the total impulse of blast wave can’t be determined by the peak overpressure. The known far asymptotic law takes no place in the flow considered at dust concentrations $\alpha_2 \equiv 3 \cdot 10^{-3}$. The shock wave with leading discontinuity disappears in dust air on finite distance from its origin and blast wave turns into dispersive wave without discontinuity.

INTRODUCTION. Point-blast problem is a classic problem in the theory of blast processes. In one-dimensional formulation it has self-similar solution (Sedov, 1946) within a range of strong blast wave (so-called “nearest asymptotycs”). At large distances from the origin, irrespectively to parentage process of initiation, the shock waves obey to certain regularities of changing of their parameters – so-called “far asymptotycs”. The main term of far asymptotics was obtained by Landau (1945), which in the case of spherical symmetry has the form:

$$\Delta p \equiv \frac{p_t - p_0}{p_0} = \frac{2\gamma}{\gamma + 1} \left( \frac{C_1}{R_1} \ln(R_t / R) \right) ; \quad V_t = \frac{2c_0}{\gamma + 1} \left( \frac{C_1}{R_1} \ln(R_t / R) \right) ; \quad L = C_3 R_t \sqrt{\ln(R_t / R)} ,$$

1) where $\Delta p$ and $V_t$ – dimensionless overpressure and air velocity at shock front, $c_0$ – velocity of sound in ambient air, $L$ – dimensionless wave length, $r_t(t)$ – shock front radius, $R_t \equiv r_t / r_1$, $r_1 = (E_0 / p_0)^{1/3}$ – dynamic radius, $E_0$ – blast energy, $C_3, R^* = \text{consts}$.

The evaluations of a blast event damage are grounded on the regularities of blast wave propagation. It is generally accepted, that the ability of blast wave to cause damages is determined by the value of the hydrodynamic head of gas stream behind shock front, which in ideal perfect gas depends uniquely on the law $\Delta p(R_t)$ of shock attenuation. The known results were obtained within the model of non-viscous non-heat-conductive gas; here the shock waves, once originated, exist ad infinitum. Propagating on a distances whatever long, and decreasing their amplitude $\Delta p$ up to whatever least values, they nevertheless keep discontinuous character with a jump parameters changing across the front.
While propagating in real environment, the discontinuity of shock wave is smoothed due to non-ideal air properties. The known investigations of the influence of viscosity and heat-conductivity on the shock wave structure are limited mainly to the consideration of stationary waves (Taylor 1910; Zeldovich 1946), which consist of relaxation zone with continuous compressive wave of width $l$, that separates initial and eventual states of gas equilibrium. However, such a stationary propagation of the wave is impossible without continuous external feedback. The latter is always necessary for compensation of internal dissipative losses, and it can not be realized in the case of a free shock propagation at a large distances. Free blast impulse propagates in the form of a compression wave of width $l$ followed by the wave of rarefaction of extent $L$, which weakens it as a result of their interaction. Condition $l \ll L$ usually satisfying: in the early stage, when $\Delta p$ is large, – due to relation $l \sim (\Delta p)^{-1}$ (Taylor 1910; Zeldovich 1946; Landau and Lifshitz, 1986), and in the far stage – due to growth of the rarefaction wave length as $L \sim R_t \sqrt{\ln(R_t/R^*)}$.

Condition $l \ll L$ allows to neglect in the total flow scale by the width $l$ of leading compressive disturbance and to consider it as a shock, similar to that within ideal gas model, taking into account, nevertheless, the non-ideal properties of continuous rarefied flow behind the shock. Then, the effect of dissipative processes in within a lengthy rarefaction wave behind the shock could essentially weaken the flow before the far asymptotics stage, and this could change the regularities (1), which are obtained for an ideal gas. However, the mentioned representation of blast wave structure can not succeed in determining the degree of this influence because of the impossibility to fit a front discontinuity in the viscous heat-conductive gas model. It is connected with non-hyperbolic character of equations of motion and absence of discontinuous solutions within the frames of this model.

**DUST INFLUENCE ON WAVE DYNAMICS.** At the same time, there exists one more internal mechanism in the real air motions, that makes this medium non-ideal, – mechanical and thermal interaction of air with a lot of tiny atmosphere aerosol particles due to essential difference in material properties of the phases. Almost similarly to the action of rheological viscosity and heat-conductivity in “pure” air, the interphase friction and heat exchange are distributed through the region of heterogeneous flow behind the shock wave front, as this flow has gradient character which itself disturbs the equilibrium of the two-phase medium. Despite a small mass aerosol particle concentration in air (less than $2 \cdot 10^{-4}$ kg/m$^3$ (Kondratyev et al. 1983), effect of heterogeneous influence on the law of blast wave motion can be essential, as a weak by strength particles’ dissipative action proceeds integrally in within the rarefied wave of large length $L$ and during large time interval, while shock wave propagates at great distances. In general, therefore, it is unknown whether it reached the asymptotics (1), or now. Already in the early stage of strong blast the additional drop of overpressure $\Delta p$ can exceed 2% at a real values of dust concentrations in one-velocity approximation (Sidorkina 1957).
With the aim to ascertain the influence of air heterogeneity on the asymptotic blast wave behavior and on its internal structure, the flow, which was initiated in dusty air by a point blast, is considered in present work until the stage, when the influence of heterogeneity becomes prevailing.

**FORMULATION OF THE PROBLEM.** Air is considered as two-phase medium – monodisperse suspension of solid quartz particles of radius \( a = 4 \mu m \) and of a small volume concentration, \( \alpha^2_{20} \ll 1 \). Investigation was carried out numerically within the frames of the model of continuum mechanics of two-phase media. Equations of one-dimensional motion of two-velocity, two-temperature mixture with non-collided phase of solid particles (Nigmatulin 1987) have the form:

\[
\begin{align*}
\frac{\partial \rho_1 x^{v^-}}{\partial t} + \frac{\partial (\rho_1 V_1 x^{v^-})}{\partial x} &= 0; \\
\frac{\partial (\rho_1 V_1 x^{v^-})}{\partial t} + \frac{\partial (\rho_1 V_1^2 + p x^{v^-})}{\partial x} &= -(\nu - 1) p x^{v^-} = F_1 x^{v^-} + \frac{3}{2} \alpha_2 x^{v^-} \frac{\partial p}{\partial x}; \\
\frac{\partial \rho_2 x^{v^-}}{\partial t} + \frac{\partial (\rho_2 V_2 x^{v^-})}{\partial x} &= F_2 x^{v^-} - \frac{3}{2} \alpha_2 x^{v^-} \frac{\partial p}{\partial x}; \\
\frac{\partial (\rho_2 V_2 x^{v^-})}{\partial t} + \frac{\partial (\rho_2 V_2^2)}{\partial x} &= Q_2 x^{v^-}; \\
\frac{\partial n}{\partial t} + \frac{\partial (nV_2 x^{v^-})}{\partial x} &= 0; \\
F &= -F_2 = -(1 - 1.5 \alpha_2) m f_{\mu}; \\
Q_2 &= n q.
\end{align*}
\]

Here \( \rho, V, u, E, p \) – volume density, velocity, internal energy, total energy and pressure; \( \alpha_2, n \) – volume concentration and number of particles in a volume unit; \( F \) – interphase force, which acts onto \( i \)-th phase; \( f_{\mu} \) – friction force, which acts on a particle; \( Q_2 \) – heat transferred to particles by means of thermal conductivity; subindex \( i = 1, 2 \) refers to carrying and dispersed phases; \( \nu = 1, 2, 3 \) is parameter of symmetry of one-dimensional flow.

Rheological friction inside carrying phase was assumed to be negligibly small, so, it was treated as non-viscous non-heat-conductive perfect gas. This made the subsystem of gas motion equations (2) hyperbolic and permitted to fit the discontinuity in gas phase. It was assumed that interphase mass transfer is absent, non-
ideal effects were taken into account only at mechanical and thermal interactions between gas and particles. For the viscous interphase force \( f_\mu \) and intensivity of thermal exchange \( q \) the following conventional expressions were used:

\[
f_\mu = \pi a^2 C_d \rho_i^0 \left| V_1 - V_2 \right| \left[ V_1 - V_2 \right] / 2, \quad C_d = \begin{cases} 24 \text{Re}^{-1} + 4 \text{Re}^{-0.33}, & 0 < \text{Re} < 700; \\ 4.3 \lg^2 \text{Re}, & 700 \leq \text{Re} < 2000; \end{cases}
\]

\[
q = 2\pi a \text{Nu} \lambda_i (T_1 - T_2) \quad \text{Nu} = 2 + 0.6 \text{Re}^{0.5} \text{Pr}^{0.33},
\]

where \( C_d \) – particle drag coefficient, \( T \) – temperature, \( \rho_i^0 \) – material (true) density of \( i \)-th phase, \( \mu \) is viscosity, \( \text{Re} = 2a \rho_i^0 \left| V_1 - V_2 \right| / \mu_i \) – Reynolds number of relative motion of phases, \( \text{Pr} = c\mu_i / \lambda_i \) – Prandtl number, \( \text{Nu} \) – Nusselt number. For the coefficients of viscosity and heat conductivity the following values were used (Nigmatulin 1987): \( \mu_i = 1.85 \cdot 10^{-5} \text{kg/ m/ sec} \), \( \lambda_i = 0.025 \text{kg/ m/ sec}^\circ \text{K} \), as well as for thermal capacity \( c_{v,i} = 716 \text{ m}^2 / \text{sec}^2 \cdot \text{K} \), \( c_2 = 710 \text{ m}^2 / \text{sec}^2 \cdot \text{K} \).

The three-stage calculation scheme (Girin 2004) was applied for numerical integration of system (2), (3), which is grounded on conservative difference schemes with adaptive grid. The gas phase subsystem (2) has been solved by the scheme, based on the method of splitting of the processes of gas motion and of interphase interaction. This gave the possibility to fit the discontinuity in compressible carrying phase and to determine jump of unknown functions across the shock. In turn, this permitted to carry out an accurate calculation of relative velocity and temperature, which determine the mechanical and thermal non-equilibrium of the flow. Thus, kinetics of interphase relaxation processes in the zone of the most non-equilibrium immediately behind the shock were reflected properly. To hold the accuracy of calculations on the stage, when the shock propagates at great distances and rarefied wave has large extent the new cells have been added in the vicinity of shock front. Numerical code was tested by means of comparison of calculations for ideal air (at \( \alpha_{20} = 0 \)) with the most precise calculations for \( v = 3 \) (Brode 1955; Okhotsymsky 1957) and showed the worst discrepancy 0.28 % in redundant pressure over the range \( R_t < 82 \). The calculation routine was approved also when some one-dimensional non-stationary problems of two-phase dispersive shock-wave flows were solved (Girin 2004; 2006; 2007).

In order to compare the results of investigation in dusty air with those in pure one, an initial values were taken in present study in accordance with work of Okhotsymsky et al. (1957). Namely, self-similar Sedov’s distributions were applied for air in initial disturbed region \( r_t0 \) at \( t_0 = 1.28 \cdot 10^{-3} \text{ sec} \) and values of definitive parameters \( \gamma = 1.4 \), \( E_0 = 8.38 \cdot 10^{13} \text{N/ m}, \quad r_{t0} = 938.81 \text{ m}, \quad p_0 = 101306.8076 \text{ Pa}, \quad p_t = 1766 \cdot 10^5 \text{ Pa}, \quad \rho_0^0 = 1.2270 \text{ kg/ m}^3, \quad c_0 = 331.57 \text{ m/ sec}, \quad T_{t0} = T_{20} = 293 \text{ K} \) were chosen.
RESULTS OF INVESTIGATION. Comparison of obtained here results for a point blast in a dusty air with those in a pure air has shown their significant difference in asymptotic behavior of the blast impulse. First of all, the wave internal structure undergoes irreversible changing under action of a dust. Relative velocity of phases, which is the measure of media mechanical interaction, initially has a maximum value at a wave front. Hence, the utmost dissipative action of particles here leads to additional (with respect to acoustic in a pure air) diminishing of jump of parameters across the shock that gradually suppresses it and eventually annihilates it. The thermal dissipative interaction enhances the suppression.

The process of shock suppression is illustrated by graphs of air velocity profile in the wave at various distances of front to the blast center (fig. 1). They were processed in accordance with asymptotic law (1): velocity was related to its front value in pure air: 

\[ Q = \frac{V}{V_f} = \frac{2c_0}{\gamma + 1} \frac{C_1}{R_f \sqrt{\ln(R_f/R')}} \]

where \( R_f \) is the distance to the characteristic value of wave length: 

\[ X = (R - R_f)/C_1 R_f \sqrt{\ln(R_f/R')} \]

When propagating in pure air, the wave profile in pointed variables is unchangeable at the stage of far asymptotics. As distinct, in dusty air the wave profile deforms fast enough just behind the front. The gas phase decelerates rapidly at the front as a result of maximum dissipative action here. Therefore, the relative velocity drops, and the maximum shifts from the front inside the wave. Eventually shock wave turns into dispersive wave which has no discontinuity and propagates with the velocity less than sound speed in ideal air \( c_0 \) (Diakov 1954). This feature of the process is general for unsupported waves in a dusty air and it is corroborated by other investigations of shock-wave flows in two-
phase dispersive medium, such as asymptotic shock wave behavior after plane gas layer breakup (Girin 2006) and the settling of a stationary wave structure at a constant velocity piston motion in dusty air (Girin 2007). The gradual integral dissipative action of a great mass of fine particles on a shock front is illustrated in fig. 2 by calculated dependences $\Delta p(R_f)$. They show, that jump of parameters across shock front in dusty air follows to another regularities, than in the case of ideal air, beginning just from the early of far asymptotics range. At dust dispersity $a = 4 \cdot 10^{-6} m$ an additional decreasing of redundant pressure, caused by the dust action, already makes up $\approx 12\%$ at $R_f = 3.0$, $\approx 60\%$ at $R_f = 10$, and $\approx 87\%$ at $R_f = 20$.

Fig. 2. Dependencies $\Delta p(R_f)$ in ideal (upper curve) and in dusty (lower curve) air (logarithmic scale); $a = 4 \cdot 10^{-6} m$, $\rho_{20} = 1.22 \cdot 10^{-2} kg / m^3$.

In ideal air the behavior of shock wave at large $R_f$ can be uniquely determined by expressions (1), if constants $C_3, R^*$ are known. This requires two measurements of wave parameters, which can be done, for example, as a result of calculations. Let $(\Delta p)_1, (\Delta p)_2$ be redundant pressures, “measured” at distances $R_{f1}, R_{f2}$. Then $C_3, R^*$ can be found as solutions of the system

$$
C_3 = \frac{\gamma + 1}{2\gamma} \frac{(\Delta p)_2 R_{f2}^2}{\ln(R_{f2}/R^*)} \left(\frac{(\Delta p)_1 R_{f1}}{\ln(R_{f1}/R^*)}\right);
$$

$$
C_3 = \frac{\gamma + 1}{2\gamma} \frac{(\Delta p)_2 R_{f2}^3}{\ln(R_{f2}/R^*)}. \left(\ln(R_{f2}/R^*)\right). \frac{\ln(R_{f1}/R^*)}{\ln(R_{f1}/R^*)}.
$$

Making use of results of most accurate and distant calculations of Okhotsymskiy et al. (1957; 1962) we obtain values $C_3 = 0.2699, R^* = 0.9463$, with $R_{f1} = 35.7508$, $R_{f2} = 82.3488$. The close values $C_3 = 0.2697, R^* = 0.9322$ with $R_{f1} = 35.4902$, $R_{f2} = 82.4005$ were obtained as a
result of present calculations. It must be noted, that value $R^* = 3.06$, which was found in (Aslanov 2006), more than three times differs from those given above.

Dependency $Z_3(R_f) = \frac{\gamma + 1}{2\gamma} \Delta p(R_f) \cdot R_f \sqrt{\ln \left( \frac{R_f}{R_{\infty}} \right)}$, that was processed as a result of present calculations in accordance with (1), is given in fig. 3. The $Z_3(R_f)$ must reach the value $C_3$, when the wave

![Graph](image)

reaches the range of far asymptotics; vise versa, proximity in values of $Z_3$ and $C_3$ manifests that the wave has reached already this range. In ideal air (upper curve) value $Z_3(R_f)$ just at $R_f \approx 7 \div 8$ is close to asymptotic value $C_3 \approx 0.270$, which shows that blast wave have come into far asymptotics range. The lower curve shows, that in a dusty air blast wave bears strong additional dissipative action of dust in a range $R_f > 2.0$, which continuously diminishes the shock amplitude. This curve clearly manifests, that blast wave in dusty air never follows asymptotic law (1).

It ought to be noted, that while processing dependence $Z_3(R_f)$ some inaccuracy was found in results of Okhotsymsky, which was admitted in calculations after methodics (Okhotsymsky 1957) was changed to that of (Okhotsymsky 1962), which occurred at $R_f = 92.934$. There is sharp perturbation in the $Z_3(R_f)$ graph at $R_f = 118.336$ (fig. 4), which exceeds 4% and get the evidence of this inaccuracy, that can not be revealed in dependence $\Delta p(R_f)$. 
THE PLANAR BLAST. In a similar way investigation was carried out for the planar case ($\nu=1$), when the blast has been modeled by breakup of plane high-pressure gas layer in a dusty air. Analysis of results in this problem have led to the same conclusions: blast wave in dusty air never follows far asymptotics law at dust densities $\rho_{20} > 10^{-3} \text{kg/m}^3$.

Dependencies $Z_3(R_f)$ in ideal air: $I$ – according to calculations (Okhotsymsky, 1957; 1962), $2$ – our data.

Fig. 4. Dependencies $Z_3(R_f)$ in ideal air: $I$ – according to calculations (Okhotsymsky, 1957; 1962), $2$ – our data.

Fig. 5. Dependencies $Z_3(t)$ ($t$ – in sec) in dusty air for $\rho_{20} = 4.08 \cdot 10^{-3}$; $1.3 \cdot 10^{-3}$; $4.08 \cdot 10^{-3}$; $1.3 \cdot 10^{-2}$; $4.08 \cdot 10^{-2}$ (kg/m$^3$) (from top to bottom). Upper curve – for ideal air.

Fig. 5. Dependencies $Z_3(t)$ ($t$ – in sec) in dusty air for $\rho_{20} = 4.08 \cdot 10^{-3}$; $1.3 \cdot 10^{-3}$; $4.08 \cdot 10^{-3}$; $1.3 \cdot 10^{-2}$; $4.08 \cdot 10^{-2}$ (kg/m$^3$) (from top to bottom). Upper curve – for ideal air.

Dependencies $Z_3(t) = z(t)\sqrt{t}$, where $z = \Delta p / \gamma p_0$, are given for various values of dust density $\rho_{20}$ in fig. 5. In ideal air $Z_3(t)$ quickly goes up to constant value $C_1 \approx 0.051$ (upper curve), which corroborate to far asymptotycs law for planar case.
At a weak dust concentrations the action of dust gradually diminishes $Z_0(t)$ from this value. In dusty air of usual concentrations, $\rho_{20} = 4 \cdot 10^{-4} \, \text{kg/m}^3$, the shock follows far asymptotic low only approximately, but at some greater values, when $\rho_{20} \approx 3 \cdot 10^{-3} \, \text{kg/m}^3$, the shock never obeys far asymptotics law. At distances of order $R_t \approx 10^3$ the **leading shock disappears, and blast wave turns by this into dispersive wave, which has no discontinuity and propagates with velocity less than the sound velocity in ideal air.**

The process of suppression of discontinuity and transformation of shock wave into dispersive wave can be illustrated by profiles of carrying phase velocity in the wave, taken at different distances, which are given in fig. 6. The general regularities of such a transformation are similar for the planar and spherical cases of symmetry.

![Fig. 6. The deformation of velocity profile in plane wave and vanishing of shock under influence of air heterogeneity at $\rho_{20} = 10^{-2} \, \text{kg/m}^3$.](image)

![Fig. 6. Changes of relative velocity profile under action of interphase friction in plane shock wave at various distances $R_t$, spent by the wave front in a dusty air.](image)
The profiles of relative velocity of phases $V_{rel}(X) = V_1 - V_2$ in the wave at different distances $R$, which were processed similarly in accordance with regularities of far asymptotics for planar case, are given in fig. 7. Similarly to the case of spherical symmetry, at the stage of strong shock wave the maximum of relative velocity in planar case is located at the shock front and gradually the dissipative action of dust suppresses discontinuity. The maximum leaves the front and follows dispersive wave, being located at the front part of it.

**CONCLUSIONS.** The investigation have shown that within the range of far asymptotics the behavior of blast wave initiated by a point blast in a dusty air differs essentially from that in an ideal air. Wave profile is gradually transforming under long-lasting dissipative action of interphase friction and heat transfer, and parameters at shock front bear additional, with respect to acoustic in the case of ideal air, decreasing. Shock wave disappears at finite distance from its origin and turns into dispersive wave without leading shock. As a result, the total impulse of the wave already can not be expressed by the value of overpressure $\Delta p$. At dust concentrations $\rho_0 \gtrsim 3 \cdot 10^{-3} \text{ kg/m}^3$ the known far asymptotic law takes no place in the point blast flow. This conclusion means that the results of conjugation of nearest and far asymptotic laws in order to obtain the regularities of overpressure changing in the practically significant zone of middle-range values $0.1 < \Delta p < 10$ (Aslanov 2006) may differ essentially from the real values. If we take into account more strong, with respect to the dust action, dissipative effect of rheological viscosity and heat conductivity of real air, the obtained here results put certain doubts concerning whether the real blast waves would achieve the known regularities of far asymptotics, which were found within the frames of ideal gas model. The real properties of air begin to effect just at the stage, when shock wave has been weakened enough by (acoustic) expansion and comes into far asymptotics range $\Delta p < 0.1$.

As the real values of a volume dust density are less, effect of a shock suppression would be less too, than that, obtained in present calculations, but this diminishing will be partially compensated by a stronger dissipative action of real particles as they have smaller sizes. Besides, if we take into account an air rheological viscosity, effect of shock suppression will be much greater, as this mechanism works similarly to the considered heterogeneous one in the zones of wave with a gradient of carrying phase velocity.

**References.**